Strategy | Games | Rivalry

Mathematics Club IITM

September 2024

Instructions

- This problem set consists of 4 questions.

- You are supposed to write your answers and upload it in the Google Form provided. You can submit a scanned copy of handwritten answers, submit a typed document or a handwritten document on a tablet explaining your answers. You can even perform a simulation analysing the various outcomes in any of the games and report your results. Anything works as long as it is legible and clear and we are able to interpret your work!

- Try to answer all the questions in a clear and readable manner and mention all your assumptions/reasons explicitly.

- You are encouraged to initiate discussions about the problems in the WhatsApp group in case you wish to discuss aspects of your strategies (Without revealing the entirety of your solution of course :))

- Keep in mind that this problem set is intended to get your brains working and has been made purely for the love of problem solving. The criteria under which we will select the 'best' solution is entirely subjective.

- Feel free to reach out to us on the official WhatsApp group for doubts!

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§1 Intra-club Humility

Post SGR, your friendly neighbourhood Mathematics Club Coordinators collect the left-over candies from the stalls and keep them in a jar. Shivanshu, Madhav and Kashyap take turns in taking from the jar of 1000 candies to share. Shivanshu goes first, Madhav goes next, and Kashyap goes last. For example, Shivanshu could take 500 candies, then Madhav could take 400, and Kashyap would take the remaining 100.

No one wants to be seen as greedy, but no one wants to end up with the fewest candies either! As such, their goals are (in order of preference):

1. Do not end up with the most candies, nor the fewest candies (a tie with most or fewest also fails this condition)

2. End up with as many candies as possible

All of them are logical, rational, know each other's goals, but cannot communicate before or during sharing. How many candies should each of them end up with?

§2 Divine Foresight

There is an island with 100 perfect logicians, each with green eyes. None of them know their own eye color, but they can see everyone else's eyes. They are not allowed to communicate with each other, except for observing others. Every night at midnight, if a logician figures out their own eye color, they must leave the island by crossing a bridge to the mainland. There is a rule that if someone knows their eye color, they must leave that very night.

One day, Maha Prasanna Guruji comes to the island and makes a single announcement: "At least one of you has green eyes." He leaves, and the islanders continue with their daily routine.

What happens now?

§3 A Twist away from Freedom

You are one of a 100 prisoners locked in individual cells. You have no sense of day or night. At random intervals, Navin, the warden summons one of the prisoners to a room with a needle resting on a table. Each prisoner must move (twist) the needle by exactly 90 degrees (1/4 rotation) in either direction. The warden may call any prisoner any number of times and no prisoner knows which other prisoners have been called before them. This process continues indefinitely and it can be assumed that at one point all prisoners are called. (In essence, you have no information about the state of the other prisoners except the state of the needle.)

At any point, if the called prisoner claims that all prisoners have visited the room at least once, they are all freed. Since Achintya convinces Navin to give you a *fair chance* to make the game more interesting, he allows you all to strategize initially before the first person is called. He also tells you that the table contains a mark at one of the four possible positions of the needle. Come up with a strategy to escape the prison. Assume the initial state of the needle is not known to the prisoners.

§4 Saturday Spiel

Deena and Pratyaksh approach you on a lovely Saturday evening and invite you to play a card game.

Deena takes out the standard deck of 52 cards, takes one of the suits (say Spades) and keeps it aside. This suit is the *prize suit*. Now he takes one entire suit (say Hearts), Pratyaksh takes another suit (say Diamonds) and you take the remaining suit (in this case Clubs). The suits that each player has acts as a kind of *currency*.

Now, Deena shuffles the suit that had been initially set aside (the *prize suit*) and takes one card from it and lays it down face-up. The remaining cards in the *prize suit* are face-down and are revealed one-by-one, each turn of the game. The game ends when there are no more cards left in the *prize suit*.

The rules for playing each turn of the game is as follows:

1. Each player must simultaneously bid for the card that is shown from the *prize suit* using the *currency cards* that they have with them.

2. Whoever bids using a *currency card* with the highest value gets to take away the *prize card*. In case of a tie amongst the highest bid, the *prize card* is discarded and no one wins it.

3. *Currency cards* that are used for bidding are discarded from the players' hands irrespective of the outcome of the turn.

Assume that each suit contains cards whose values are 1, 2, 3, ..., 13 in some fixed fashion. For example you may assign the value 1 to Ace, 2 to the card numbered 2, and so on. The order in which values are assigned doesn't

matter since each player starts off with the same set of cards.

At the end of the game, whoever has the highest total value in the *prize cards* that they have won gets a Coolbiz treat from Navin. Given this information and assuming you like Coolbiz treats, what strategy would you use to play this game?

Note that there may not be any definite answer to this question as your strategy may depend on the strategy that the other players adopt. You can assume that Deena and Pratyaksh are simply playing randomly and don't have any particular inclination towards a Coolbiz treat or you can assign some strategy to their actions and then come up with your strategy. I am leaving it open to you to make any assumptions that you wish, but do state them in your answers!