§1 Bronze

Find all four digit numbers having the following properties:

1. it is a square
2. its first two digits are equal to each other
3. its last two digits are equal to each other

§2 Silver

Navin picks real numbers X and Y are at random from the interval $(0, 1)$. Compute the probability that the closest integer to X/Y is even.

§3 Gold

During an intense game of foosball, Achintya has a constant probability of 0.4 of making any given shot, independent of previous shots. Let a_n be the ratio of shots made to shots attempted after n shots. The probability that $a_{10} = 0.4$ and $a_n \leq 0.4$ for all n such that $1 \leq n \leq 9$ is given to be $p^a q^b r / (s^c)$ where p, q, r, and s are primes, and a, b, and c are positive integers. Find $(p + q + r + s)(a + b + c)$.

§4 Silver

Two squares of a 7×7 checkerboard are painted red, and the rest are painted blue. Two color schemes are equivalent if one can be obtained from the other by applying a rotation in the plane board. How many inequivalent color schemes are possible?

§5 Silver

Choose at random seven points on the circle $x^2 + y^2 = 1$. Interpret them as cuts that divide the circle into seven arcs. Compute the expected length of the arc that contains the point $(1, 0)$.

§6 Bronze

Given a rational number, write it as a fraction in lowest terms, i.e., $\frac{p}{q}$ where p, q are co-prime and calculate pq. For how many rational numbers between 0 and 1 will 20! be the resulting product?