§1 Silver

Let a and b be integers such that $a + b = 2025$. Find ($\det(A)$ - 2) mod 23 where the matrix A is

$$\begin{bmatrix} a^3 & b^3 & 3ab & -1 \\ -1 & a^2 & b^2 & 2ab \\ 2b & -1 & a^2 & -b^2 \\ 0 & b & -1 & a \end{bmatrix}$$

§2 Bronze

Given that A and B are square matrices of the same order with rank 3 and 2, and $B^T A = 0$, then what is $rank(A - B)$?

§3 Silver

How many real 5×5 matrices A $= (a_{ij})$ exist such that

$$|a_{ii}| > \sum_{j \neq i} |a_{ij}|$$

for all $1 \leq i \leq n$ and $\det(A) = 0$? Report the answer mod 5.

§4 Bronze

What is the determinant of the following matrix?

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 2 & 5 & 7 \\ 9 & 4 & 25 & 49 \\ 27 & 8 & 125 & 343 \end{bmatrix}$$

§5 Silver

A_n is a $n \times n$ matrix such that $a_{ij} = \gcd(i, j)$. Find $\det(A_{10})$

Hint: Find $\dfrac{\det(A_n)}{\det(A_{n-1})}$

§6 Gold

Let A $= (a_{ij})$ be a square matrix of order 16. Suppose $a_{ij} = \binom{16i}{j}$ Evaluate its determinant.