§1 Silver

Let P be an interior point in $\triangle ABC$. The cevians AP, BP, CP meet the sides BC, CA, AB at D, E, F respectively. Given that the lengths $AB = 51, BC = 31, AC = 42, AF = 20, BD = 11$, find $\dfrac{AP}{37 \times PD}$.

§2 Bronze

Consider $\triangle ABC$, which is acute-angled, and let CD be the altitude to AB. The lengths $AB = 8, CD = 7$. Find the distance between the midpoints of AD and BC.

§3 Silver

Consider a square $WXYZ$ inscribed in a circle. Another square $ABCD$ is positioned such that vertices A and B lie on side $\overline{YZ}$ of square $WXYZ$, while vertices C and D lie on the circle.

Given that the area of square $WXYZ$ is 1, determine the area of square $ABCD$ in the form $\dfrac{m}{n}$, where m and n are relatively prime positive integers with $m < n$. Compute the value of $10n + m$.

§4 Gold

P lies in the interior of triangle ABC. X, Y, Z are the feet of the perpendiculars from P to the lines BC, CA, AB respectively. Its given to you that P is the point for which

$$\frac{BC}{PX} + \frac{CA}{PY} + \frac{AB}{PZ}$$

is least. Now consider ABC to have sides 13, 14 and 15. Find the value of $PA \times PB \times PC$.

§5 Silver

You start with a right triangle $\triangle ABC$ where $AB = 90$, $AC = 120$, and $BC = 150$, and you inscribe a circle C_1. Nice and clean. But then, chaos. You construct $\overline{DE}$ (perpendicular to $\overline{AC}$, tangent to C_1) and $\overline{FG}$ (perpendicular to $\overline{AB}$, also tangent to C_1).

The universe retaliates: let C_2 and C_3 be the incircles of $\triangle CDE$ and $\triangle BFG$. The distance between their centers is $\sqrt{10n}$. Your punishment: Find n.

§6 Bronze

ABCD is a cyclic quadrilateral such that $AC \perp BD$. AC intersects BD at E. Given that ABCD is always inscribed in a circle of radius 5, for all such arrangements, say the value of

$$EA^2 + EB^2 + EC^2 + ED^2$$

is at most b and at least a. Find the value of $b - a$.