

§1 Bronze

Find the maximum and minimum values of $f(x, y, z) = x^2 y^2 z$ in the region $\{(x, y, z) | x^2 + 2y^2 + 3z^2 \leq 1\}$

§2 Silver

Let the value of the summation

$$\sum_{a=1}^{\infty} \sum_{b=1}^{\infty} \sum_{c=1}^{\infty} \frac{1}{abc(a+b+c)}$$

be x . Find the value of $\frac{x}{\alpha}$ where $\alpha = \sum_{n=1}^{\infty} \frac{1}{n^4}$

§3 Silver

Let $\alpha_1, \alpha_2, \dots, \alpha_{10}$ be real numbers such that:

$$\alpha_1 + \alpha_2 + \dots + \alpha_{10} \geq 10$$

and

$$\alpha_1^2 + \alpha_2^2 + \dots + \alpha_{10}^2 \geq 100.$$

Define: $k = \max(\alpha_1, \alpha_2, \dots, \alpha_{10})$. Find the minimum possible value of k .

§4 Silver

Let $f: \mathbb{N} \rightarrow \mathbb{N}$ be a function such that:

$$f(n+1) > f(n) \quad \text{for all } n \in \mathbb{N}$$

and

$$f(f(n)) = 3n \quad \text{for all } n \in \mathbb{N}.$$

Evaluate $f(2025)$

§5 Gold

$$f(n) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} e^{-x^T A x} dx_1 dx_2 \dots dx_n$$

where,

$$A_{n \times n} = \begin{bmatrix} 2 & -1 & 0 & 0 & \dots & 0 \\ -1 & 2 & -1 & 0 & \dots & 0 \\ 0 & -1 & 2 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \\ 0 & 0 & 0 & \dots & 2 & -1 \\ 0 & 0 & 0 & \dots & -1 & 2 \end{bmatrix} \quad \text{and} \quad x^T = [x_1 \quad x_2 \quad \dots \quad x_n]$$

Find $\lim_{n \rightarrow \infty} (f(n))^{\frac{1}{n}}$

§6 Silver

Find the sum of the volume and surface area of the 6-dimensional hyper-sphere of unit radius.