Question 1



$$\alpha = \int_0^1 x e^{\frac{x^2 - 1}{2}} \cos(x) dx$$

$$\beta = \int_{1}^{3/2} e^{2(x^2 - 2x)} \sqrt{1 - \cos(4x - 4)} dx$$

Find
$$\frac{\alpha - \cos(1) + e^{-1/2}}{\beta}$$

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Question 2



$$\psi(x) = \lim_{n \to \infty} \int_0^x \sum_{k=1}^n \frac{1}{2^k (1 + t^k)} dt$$

If
$$\psi(x) = \sum_{n=0}^{\infty} a_n x^n$$
 for $|x| < 1$, then find the value of $a_0 + a_1 + a_2$

Question 3



$$f(x,y):=\frac{x}{x^2+y^2} \text{ and } g(x,y):=\frac{-y}{x^2+y^2}$$

$$P(x,y) = \int_0^y \frac{\partial f(x,t)}{\partial x} \partial t - g(x,y)$$

$$Q(x,y) = \int_0^x \frac{\partial g(t,y)}{\partial y} \partial t - f(x,y)$$

Find
$$\int_{1}^{2} \left(\frac{\partial P}{\partial x} \right) \partial y + \int_{2}^{3} \left(\frac{\partial Q}{\partial y} \right) \partial x$$

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Question 4



$$\int_{1}^{2} \ln x \left(\left(\frac{x}{e} \right)^{x} + \left(\frac{e}{x} \right)^{x} \right) dx$$

Question 5



$$\int_0^1 \left(1 + \frac{x^5}{5!} + \frac{x^{10}}{10!} + \dots \right) dx$$

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Question 6



$$\int_0^\infty e^{-x^2} \cos(5x) dx$$

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$$\int_0^1 \sqrt{\frac{x}{1-x}} \ln\left(\sqrt{\frac{x}{1-x}}\right) dx$$

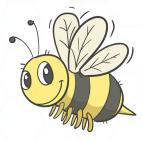
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Question Buffer R3 8



Find the value of

$$\int_0^{\frac{\pi}{4}} \left(\frac{x^{10}}{1 + x^6} \right)^2 dx$$

Question Buffer R3 9



Find the value of

$$\int \log(x) \sin^{-1}(x) dx$$

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Question Round 4



$$\int (9x^9 - x^{90} + 9x^{99} - x^{900} + 9x^{909} - x^{990} +$$
$$9x^{999} - x^{9000} + \dots) dx$$

Question Round 4



Find the value of

$$\int_0^\infty \frac{\log(2x)}{\sqrt{x+1}\sqrt{x}\sqrt{2x+1}} dx$$

Given that
$$\beta(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx$$

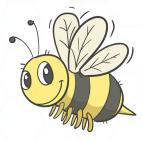
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Question Round 4



$$\int_0^\infty \ln(1 - e^{-x}) \, dx$$

Question Round 4



$$\int e^{\cos x} \cos(2x + \sin x) \, dx$$

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Question buffer



If
$$f(x, y) = \sin(x + \pi/2) + \cos(y)$$
 is

a function defined over $x \geq 0$, $y \geq 0$, $x + y \leq \pi$,

The intersection of f(x,y) and the plane $y=(\tan\theta)x \text{ is defined as } F(x,\theta) \text{ on that plane}$

$$\forall \ \pi/2 \geq \theta \geq 0$$
 Find
$$\int_0^{\pi/2} \int_0^2 F^{-1}(x,\theta) dx d\theta$$