



Integration Bee



Prelims Question Paper



MATHEMATICS CLUB IITM

September 4, 2024

Instructions

- You will be given 45 minutes to solve the 20 questions in this paper.
 - Any use of online resources / gadgets is prohibited.
 - Use of calculators of any kind is prohibited.
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1) $\int_0^1 \frac{1}{1+x^3} dx$

2) $\int_0^1 \frac{1}{1+x^3} dx + \int_0^1 \frac{y^3}{y^2+y^5} dy$

3) $\int_0^1 \frac{x^7-1}{\ln x} dx$

4) $3 \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{dx}{\cos^2 x (1 - \tan x)^2}$

5) $I_n = \int_0^1 (1-x^{24})^n dx$

Find the value of $\frac{I_{26}}{I_{27}}$.

6) $\int_0^1 \frac{e^{x^4} x^{11}}{(x^8 + 2x^4 + 2)^2} dx$

7) $\int_0^{\frac{\pi}{4}} \left(\frac{3 \sin^2 x}{1 + \tan^2 x} - \frac{1}{\sec^4 x} \right) dx$

8) Solve: $2(dy - dx) + e^{\ln(y-1)} dy = ydx - xdy$; $y(1) = -1$.

Find the value of $\int_2^6 (x - 1) dy$.

9) $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x) e^x \sec y$; $y(0) = 0$

Find the value of $\int_0^1 \sin y dx$.

10) $\phi(x) = \int \left(\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \right) \exp \left(a \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \right) da$

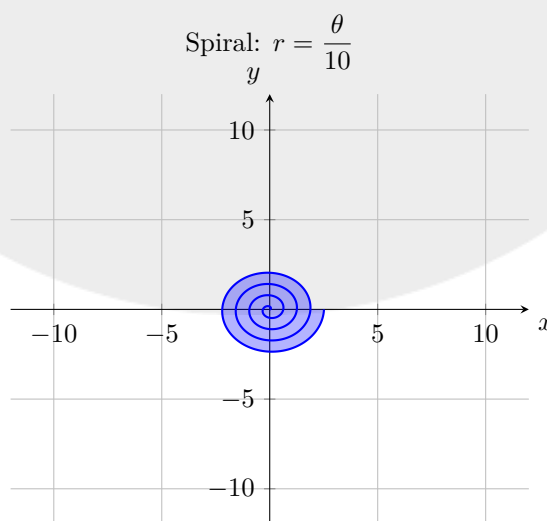
Find the value of $\phi(\pi) - \phi(0)$.

11) $\int_0^1 \frac{(\sin^{-1} x)^{2024} (\cos^{-1} x)^{2024}}{\sqrt{1-x^2}} dx$

12) $\int_0^\pi \ln(1 + \cos(x)) dx$

13) $\lim_{n \rightarrow \infty} \frac{\log_{10} \prod_{k=1}^n (100n + k)}{n} - \log_{10} n$ (Answer in terms of \log_{10})

14) Find the (shaded) area of the spiral below.



15) $\int_{-\infty}^{\infty} e^{-ex^2} e^{-ieix} dx$

Beehive Name:

Name of Bee 1:

Roll no. of Bee 1:

Contact of Bee 1:

Name of Bee 2:

Roll no. of Bee 2:

Contact of Bee 2:

1.

11.

2.

12.

3.

13.

4.

14.

5.

15.

6.

16.

7.

17.

8.

18.

9.

19.

10.

20.

