Bids & Brains
Round 1 Question Paper

Mathematics Club

February 2025

Instructions

- You will be given 30 minutes to solve the 10 questions in this paper.

- Any use of online resources / gadgets is prohibited.

- Use of calculators of any kind is prohibited.

§1 Closed mysteries

KK and KV have been approached by a mysterious individual. The individual has with him three unmarked, opaque envelopes. One contains Rs. 100, one contains Rs. 200 and the third contains Rs. 300. They are shuffled well and then one envelope is given to KK and another is given to KV (the third one is set aside). KK opens his envelope and checks its content without showing it to KV. Then he either says "pass", in which case each of them get to keep their envelope, or he asks KV to trade envelopes. KV is not allowed to see the content of his envelope and has to say either Yes or No. If he says No, then the two get to keep their envelopes. If, on the other hand, he says Yes, then they trade envelopes and the game ends. KK and KV are selfish, greedy and risk neutral. Will KV agree to trade? Give a brief explanation as well.

§2 Fair games

Deena and Navin spend an evening playing the following perfect-information game. Deena starts by choosing a number from the set $\{1, 2, 3, 4, 5, 6, 7\}$, then Navin chooses a number from this set, then Deena again, and so on. The first player who brings the cumulative sum of all the numbers chosen (up to and including the last one) to 48 or more wins. One of the two has a winning strategy. Find out who that is along with a brief explanation.

§3 Dangeous antics

Pratyaksh has gone crazy and is fond of throwing glasses belonging to Pranjal's house. A glass breaks if its thrown from above a threshold floor of a 100-floor building. Every time a glass is dropped, it is counted as an attempt. To determine the threshold floor, Pranjal gives Pratyaksh 2 glasses. With how many minimum number of attempts can Pratyaksh determine the threshold floor?

§4 Hugely small

Aditi is intrigued by number sequences and is playing with an array of numbers. Pranjal gives her an array of n numbers. To find the minimum number, it takes $n - 1$ comparisons. To find the maximum, it takes $n - 1$ comparisons. Aditi wants to simultaneously find both minimum and maximum, what are the minimum number of comparisons she needs to make to be able to find them?

§5 Tumultuous walls

Pranjal and Shivanshu play a game in which they take turns removing one brick or two adjacent bricks from one "wall" among a set of several walls of bricks, with gaps possibly creating new walls. The walls are one brick tall. For example, a set of walls of sizes 4 and 2 can be changed into any of the following by one move: (3, 2), (2, 1, 2), (4), (4, 1), (2, 2), or (1, 1, 2).

Pranjal plays first, and the player who removes the last brick wins. Consider (6, 2, 1) to be the starting configuration. If both players play optimally who wins?

§6 A cat & mouse chase?

A clever individual, KK, is attempting to escape from Prad, who is blind but determined to catch him. To aid his pursuit, Prad gifts KK a watch with a motion detector. However, KK, being astute, positions his arm perpendicular to his direction of movement at a fixed distance of 1 meter from his body, thereby misleading the detector. The movement follows these rules in the sequence :

- KK moves exactly 1 meter directly away from Prad's position at each step.

- Prad moves exactly 1 meter toward the direction indicated by the detector, which is based on KK's deceptive arm position.

Initially, both KK and Prad start at the same position, i.e., d(0) = 0. Let $d(n)$ denote the distance between Prad and KK after n steps. Compute the following limit:

$$\lim_{n \to \infty} (d(n) + 1)^{\frac{1}{n}}.$$

§7 Flipping Coins

Atreya and Pradyumnan place a game with coins. Atreya tosses his $n + 1$ fair coins and Pradyumnan tosses his n fair coins. The coin tosses are independent. Now after all the $2n + 1$ coin tosses what is the probability that Atreya has gotten more heads than Pradyumnan.

§8 Wins by 100 points!

Consider a trapezoid in which one base is 100 cm longer than the other. A segment is drawn to join the midpoints of its legs, dividing the trapezoid into two regions with an area ratio of 2 : 3. Now, we seek to draw another segment, parallel to the bases and joining the legs, such that it divides the trapezoid into two equal-area regions. Determine $\lfloor x^2/100 \rfloor$, where x is the length of this segment.

§9 As simple as that∼

Consider $\triangle ABC$, where E is a point on AC and BE is the angle bisector of $\angle B$. Suppose $\angle A = 2\angle C$ and $CE = AB$. Find $\angle B$.
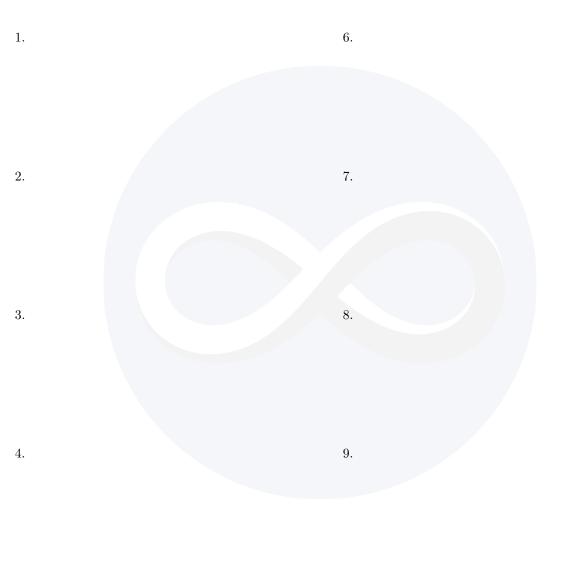
§10 Well Determined!

The entries of a 3×3 matrix A are $+1$ or -1, what is the probability that the determinant of A is negative?

Team Name:

Name of Bidder 1:
Roll no. of Bidder 1:
Contact of Bidder 1:

Name of Bidder 2:
Roll no. of Bidder 2:
Contact of Bidder 2:

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.