Mathematics Club
Contingent Problem Set - 8

Challenge posed on: 02/08/2024 **Challenge conquered by: 09/08/2024**

1 Overview

- **Topics focused:**
  - Geometry
  - Geometric Inequalities
- **Challengers:**
  - Anjali
  - Karthikeya
  - Naveen Koushik
- Typical color coding
- Few problems are standard problems the idea is to revise basics and get familiar with standard configurations. Refrain from googling as much as possible.
- If you can draw neat and correct diagrams more than half the problem is solved.

2 Problems

1. **Nice name for the problem** A convex hexagon $ABCDEF$ is inscribed in a circle such that $AB = CD = EF$ and diagonals AD, BE, CF are concurrent. Let P be the intersection of AD & CE. Prove that $\frac{CP}{PE} = \left(\frac{AC}{CE}\right)^2$

2. **Back to Basics**

   (a) Let S and H be the circumcenter and the orthocenter of the $\triangle ABC$ respectively. It is known that BH is the angluar bisector of $\angle ABS$. The line passing through S and parallel to AB meets AC at K. Then show that $AH = AK$.

   (b) Let Γ be the incircle of a scalene $\triangle ABC$. Let r_A be the radius of the circle which is tangent to Γ and the sides AB and AC. Define r_B and r_C similarly. Then find the radius of Γ in terms of r_A, r_B and r_C.

   (c) In $\triangle ABC$, let P and R be the feet of the perpendiculars from A onto the external and internal bisectors of $\angle B$ respectively and let Q and S be the feet of the perpendiculars from A onto the internal and external bisectors of $\angle C$ respectively. If $PQ = 7, QR = 6$ and $RS = 8$. What is the $[\triangle ABC]$?

3. **Geometry?** A $1 \times d$ rectangle is cut by two perpendicular lines into 4 smaller rectangles, three of them have areas not less than 1. While the area of the fourth one is not less than 2. Find the smallest positive number d for which this is possible.

4. **Kill Bill Pandey!**

   (a) Eleven soldiers are standing on a battleground and distance between them are all distinct.When the bell starts chiming, each of them shoots at the one among the other nine soldiers who is the nearest and kills him. Find out atleast how many soldiers will be killed.

   (b) A king named Naveen has 1980 soldiers where each one is at atleast 1 unit apart from the others. These soldiers fight against themselves if they are less than $\sqrt{3}$ units away. Show that Naveen can choose a set of 248 soldiers so that no two of them fight with each other.

5. **Can you do without Trig Bash?** Let $AB = AC$ in $\triangle ABC$. Let P be a point on BC and Q be a point on AP such that $\angle CAB = \angle CQP = 2\angle BQP$. Then show that $2BP = PC$.
If you solve this with trig bash you get one point if you solve by pure geometry you get 10 points.

6. **Simply Beautiful** Let D be the midpoint of BC in $\triangle ABC$ and M is the foot of perpendicular from C onto AD. The circumcirle of $\triangle ABM$ intersects BC at K and R is the midpoint of AK then show that $BR = RC$.

7. **Explore as much as you can** The idea of this particular question is to make you aware of different kind of interesting geometric inequalities, maximal and minimal configurations.

   (a) Read about Steiner's triangle problem and Toricelli's Point and try to come up with your own proof in case if you couldn't read the proof from somewhere and try to explain it in your own words.

   (b) Try to come up with your own variants that are extension for these problems and solve them.

   (c) Inscribe a quadrilateral of minimal perimeter in a given rectangle.